Best practices for your confirmatory factor analysis

A JASP and lavaan tutorial

Pablo Rogers

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This supplementary material provides the complete R/lavaan implementation for the confirmatory factor analysis (CFA) tutorial presented in Rogers (2024). The analyses use the WHOQOL-BREF instrument — a 24-item quality of life measure covering four domains: Psychological, Physical, Social, and Environment — with data from a Brazilian sample of 1,047 respondents (Rogers, 2021). The codes that generates the outputs for this tutorial article can be found in the notebook: https://phdpablo.github.io/cfa-brm/index-preview.html

1 Introduction

A central challenge in applied CFA is that items are typically measured on ordinal Likert scales, yet many textbooks and software default to Maximum Likelihood (ML) estimation, which assumes continuous, multivariate normal data. As discussed in Rogers (2024), the appropriate approach for ordinal data is the Diagonally Weighted Least Squares (DWLS) estimator — implemented as WLSMV in lavaan — which operates on polychoric correlations rather than Pearson correlations and does not assume normality of observed variables.

> ❗ Tutorial Structure
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> This document is organized as a manuscript project: the **Article** presents the analytical narrative and results without showing code, while the **Article Notebook** displays the full R code behind every result. Both views share the same content — the only difference is code visibility.

2 Setup

We begin by loading all required R packages. The `lavaan` package provides the core CFA estimation engine; `semTools` extends it with reliability functions and additional diagnostics; `semPlot` produces path diagrams; `psych` provides descriptive statistics and omega coefficients for bifactor models; `simsem` enables Monte Carlo simulation for power analysis; and `dynamic` computes the Dynamic Fit Index (DFI) cutoffs.

We set global options and a seed for reproducibility. The seed ensures that all simulation-based results (power analysis) are exactly reproducible.

> **i A Note on Code Design**
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> Throughout this tutorial, you will notice that some code patterns are repeated across sections — for example, extracting fit indices, formatting factor loadings, and building comparison tables. In a production environment, these routines would typically be modularized into reusable R functions and sourced from separate scripts.
> We deliberately chose **not** to do this. Each analysis section is self-contained, with all code written explicitly and in sequence. This design sacrifices automation and conciseness in favor of **transparency**: the reader can follow each step without navigating between files or tracing function definitions. For a tutorial whose primary goal is to teach CFA best practices, we believe clarity of exposition is more valuable than code elegance.
> Readers who wish to adapt this workflow for their own research are encouraged to refactor repeated patterns into functions and organize them into modular scripts — this is, in fact, a best practice for reproducible research projects.

3 Step 1: ETL — Extract, Transform, Load

The WHOQOL-BREF dataset is loaded directly from the Mendeley Data repository (Rogers, 2021). The data file contains N = 1,047 observations with 24 items. A separate label file provides the variable names following the naming convention: item number followed by a suffix indicating its domain (_P = Psychological, _F = Physical, _S = Social, _A = Environment). Items Q3, Q4, and Q26 have already been reverse-coded in the source data so that higher values consistently indicate better quality of life.

Table 1 shows the first six observations (n). All variables are integer-coded (1–5), reflecting the five-point Likert response format.

Table 1: First rows of the WHOQOL-BREF dataset

	n1	n2	n3	n4	n5	n6
Q3_F	5	5	5	5	3	2
Q4_F	4	5	4	5	4	4
Q10_F	2	3	3	5	3	4
Q15_F	5	3	4	5	4	4
Q16_F	3	3	5	5	4	4
Q17_F	4	3	2	4	4	4
Q18_F	3	2	2	5	4	4
Q5_P	3	3	4	4	3	3
Q6_P	3	2	4	5	5	4
Q7_P	2	2	2	5	3	3
Q11_P	2	3	4	5	2	4
Q19_P	2	3	3	4	4	4
Q26_P	4	4	2	5	4	4
Q20_S	4	2	4	4	4	4
Q21_S	2	3	5	5	5	2
Q22_S	2	2	5	5	4	4
Q8_A	4	3	3	4	2	3
Q9_A	3	2	2	4	2	3
Q12_A	3	2	3	5	3	2
Q13_A	4	2	4	5	3	4
Q14_A	3	3	3	4	2	3
Q23_A	2	2	5	5	5	4
Q24_A	2	4	5	3	3	2
Q25_A	2	2	5	5	3	4

Table 2 presents the descriptive statistics for all 24 items. Means range from approximately 3.0 to 4.0, with standard deviations around 0.8–1.1, indicating moderate variability across items. No floor or ceiling effects are evident, and skewness values are generally within acceptable bounds ($|\text{sk}| < 1$), supporting the use of polychoric correlations. These statistics are consistent with the original validation study (Rogers, 2024).

Table 2: Descriptive statistics for all WHOQOL-BREF items

	vars	n	mean	sd	me-dian	trimmed	mad	min	max	range	skew	kurto-sis	se
Q3_F	1	1047	4.17	0.93	4	4.31	1.48	1	5	4	-0.99	0.27	0.03

Table 2: Descriptive statistics for all WHOQOL-BREF items

	vars	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtosis	se
Q4_F	2	1047	4.10	1.01	4	4.26	1.48	1	5	4	-1.02	0.31	0.03
Q10_F	3	1047	3.61	0.91	4	3.65	1.48	1	5	4	-0.18	-0.43	0.03
Q15_F	4	1047	4.42	0.79	5	4.57	0.00	1	5	4	-1.52	2.41	0.02
Q16_F	5	1047	3.50	1.03	4	3.49	1.48	2	5	3	-0.13	-1.16	0.03
Q17_F	6	1047	3.72	0.92	4	3.78	0.00	2	5	3	-0.51	-0.53	0.03
Q18_F	7	1047	3.75	0.96	4	3.81	1.48	2	5	3	-0.50	-0.64	0.03
Q5_P	8	1047	3.29	0.82	3	3.35	1.48	1	5	4	-0.44	-0.09	0.03
Q6_P	9	1047	3.90	0.99	4	4.02	1.48	1	5	4	-0.88	0.48	0.03
Q7_P	10	1047	3.61	0.86	4	3.66	1.48	1	5	4	-0.53	0.12	0.03
Q11_P	11	1047	3.82	0.99	4	3.92	1.48	1	5	4	-0.62	-0.09	0.03
Q19_P	12	1047	3.72	0.95	4	3.77	1.48	2	5	3	-0.42	-0.70	0.03
Q26_P	13	1047	3.91	0.92	4	4.03	0.00	1	5	4	-1.19	1.68	0.03
Q20_S	14	1047	3.64	0.89	4	3.67	1.48	2	5	3	-0.25	-0.66	0.03
Q21_S	15	1047	3.46	1.00	4	3.45	1.48	2	5	3	-0.05	-1.09	0.03
Q22_S	16	1047	3.58	0.88	4	3.60	1.48	2	5	3	-0.16	-0.69	0.03
Q8_A	17	1047	2.94	0.98	3	2.97	1.48	1	5	4	-0.11	-0.43	0.03
Q9_A	18	1047	3.47	0.91	4	3.52	1.48	1	5	4	-0.64	0.26	0.03
Q12_A	19	1047	3.45	1.10	3	3.50	1.48	1	5	4	-0.25	-0.54	0.03
Q13_A	20	1047	3.93	0.82	4	3.98	1.48	1	5	4	-0.57	0.15	0.03

Table 2: Descriptive statistics for all WHOQOL-BREF items

	vars	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtosis	se
Q14_A	21	1047	3.34	0.96	3	3.34	1.48	1	5	4	-0.29	-0.26	0.03
Q23_A	22	1047	3.98	0.97	4	4.09	1.48	2	5	3	-0.70	-0.45	0.03
Q24_A	23	1047	3.73	0.97	4	3.79	1.48	2	5	3	-0.40	-0.79	0.03
Q25_A	24	1047	3.83	0.98	4	3.91	1.48	2	5	3	-0.50	-0.72	0.03

Data Overview

The dataset contains N = 1047 observations on 24 variables. All items are measured on a 5-point Likert scale (1–5), representing ordinal data. This ordinal nature is fundamental: it requires the use of polychoric correlations and DWLS estimation rather than Pearson correlations and ML.

4 Population Model

Before fitting CFA models to our sample data, we define a *population model* that serves two purposes: (1) computing Dynamic Fit Index (DFI) cutoffs tailored to this specific model structure and sample size, and (2) conducting *a priori* power analysis via simulation.

The population model is based on the meta-analytic factor loadings from Lin & Yao (2022), who validated the WHOQOL-BREF factor structure using a meta-analysis of exploratory factor analyses combined with social network analysis. Factor correlations were set at 0.3, reflecting the upper bound reported in Lin & Yao (2022) (range 0.08–0.30). Residual variances were derived analytically as $1 - {}^2$ (except for Q3 and Q4, whose residuals are adjusted for their predicted correlation).

Given the ordinal nature of the variables, we assume equidistant thresholds following a linearity assumption with proportions of approximately 12%, 23%, 31%, 23%, and 12%. For a five-point Likert scale, this translates to threshold values of -1.2, -0.4, 0.4, and 1.2. While it is unlikely that thresholds from previous studies will be available, if response frequencies per category per item are provided, thresholds can be estimated using the inverse normal distribution.

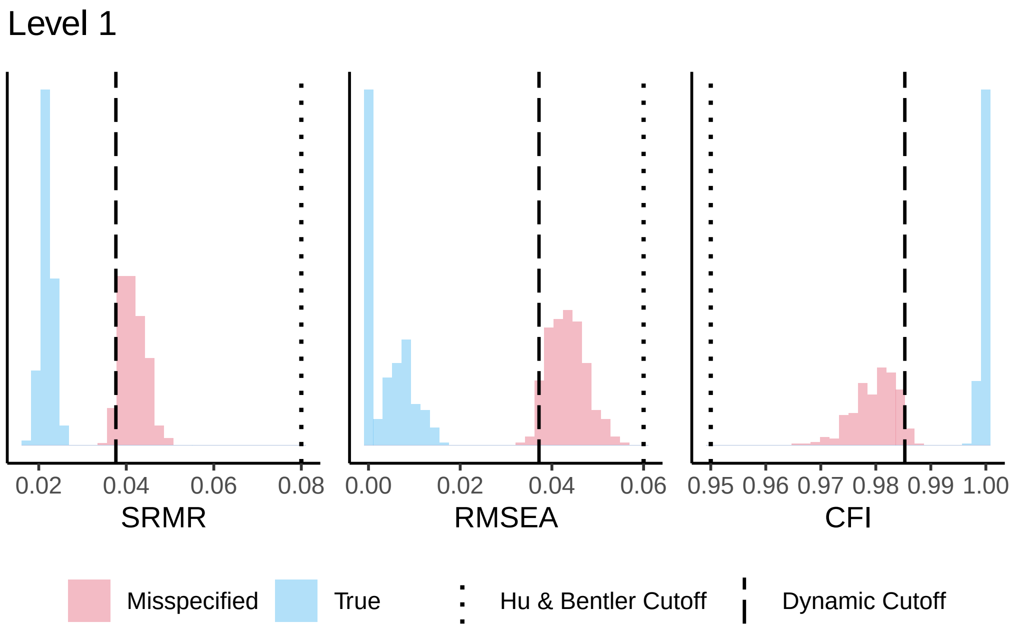
! Important Caveat

This population model should be understood as a **teaching illustration**. The loadings from Lin & Yao (2022) come primarily from exploratory factor analysis studies using principal components with varimax rotation — a method that has well-known limitations (Rogers, 2022). Ideally, one should seek a robust national survey that has employed CFA with ordinal data for the WHOQOL-BREF, providing more appropriate starting values.

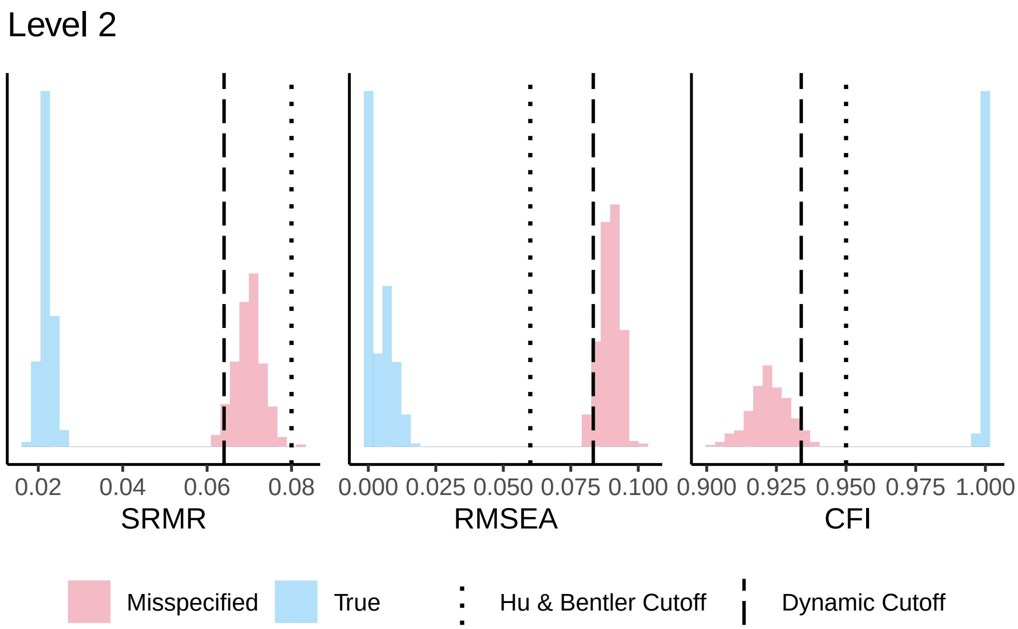
4.1 Dynamic Fit Index (DFI)

The Dynamic Fit Index (McNeish, 2023) provides simulation-based cutoff values for fit indices that are tailored to a specific model structure and sample size, rather than relying on generic rules of thumb (e.g., CFI > .95, RMSEA < .06). The DFI computation requires the population model defined above.

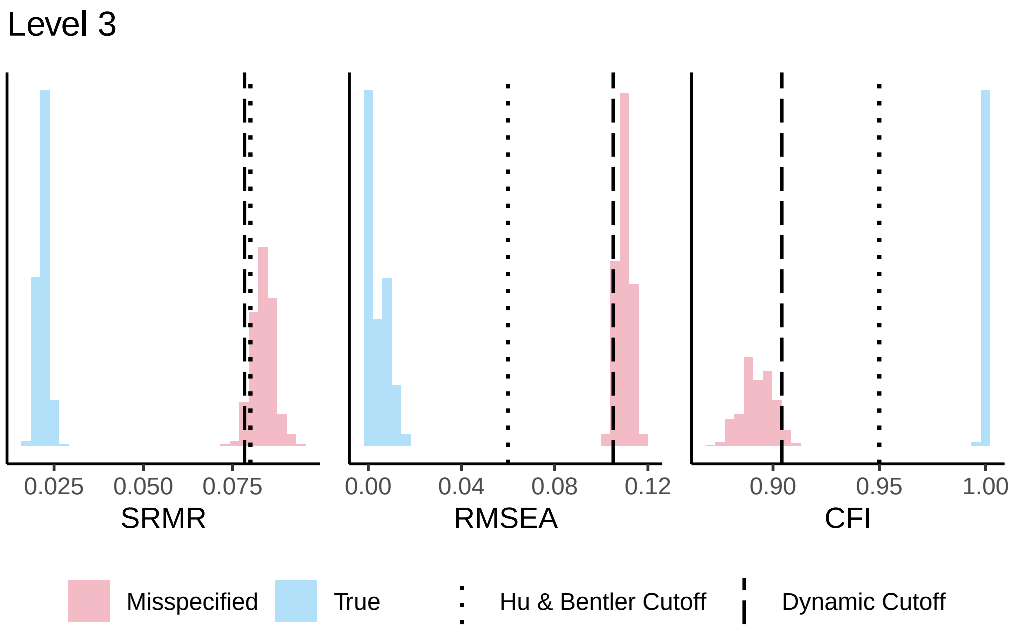
```
Your DFI cutoffs:
            SRMR  RMSEA CFI   Magnitude
Level-0     0.024 0.012 0.998 NONE
Specificity 95%   95%   95%

Level-1     0.038 0.037 0.985 0.37
Sensitivity 95%   95%   95%

Level-2     0.064 0.083 0.934 0.599
Sensitivity 95%   95%   95%

Level-3     0.078 0.105 0.904 0.58
Sensitivity 95%   95%   95%

 Notes:
  -Number of levels is based on the number of factors in the model
  -'Sensitivity' is % of hypothetically misspecified models correctly identified by cutoff i
  -Cutoffs with 95% sensitivity are reported when possible
  -If sensitivity is <50%, cutoffs will be supressed

 The distributions for each level are in the Plots tab
[[1]]
```


Level 1
0.02
0.04
0.06
0.08
SRMR
0.00
0.02
0.04
0.06
RMSEA
0.95
0.96
0.97
0.98
0.99
1.00
CFI
Misspecified
True
Hu & Bentler Cutoff
Dynamic Cutoff

[[2]]


Level 2
0.02
0.04
0.06
0.08
SRMR
0.000
0.025
0.050
0.075
0.100
RMSEA
0.900
0.925
0.950
0.975
1.000
CFI
Misspecified
True
Hu & Bentler Cutoff
Dynamic Cutoff

[[3]]



Note that we could compute a separate DFI for each estimated model (assuming each in turn as the population model). However, this would sacrifice comparability across models. Moreover, DFI methods for bifactor models are not yet available, and those for hierarchical models are still in development.

5 4-Factor CFA Model

The first model tested is the standard four correlated factor model, which represents the theoretical structure of the WHOQOL-BREF (Lin & Yao, 2022).

5.1 Model Specification

Each of the 24 items loads on exactly one of the four latent factors: Psychological (6 items), Physical (7 items), Social (3 items), and Environment (8 items). Latent factors are allowed to freely correlate, reflecting the expected interrelations among quality of life domains. A residual covariance between items Q3 and Q4 is included based on prior theoretical and empirical evidence, as both items assess related aspects within the Physical domain.

5.2 Model Estimation

The estimation uses DWLS (WLSMV in lavaan) with all items declared as ordinal (`ordered = TRUE`), and factor variances are fixed to 1 (`std.lv = TRUE`) for identification.

5.3 Model Evaluation

How to Read CFA Output

When evaluating a CFA model, watch for: (1) **Heywood cases** — standardized loadings > 1 or negative variance estimates, which indicate estimation problems; (2) **Overall fit** — SRMR $< .08$, RMSEA $< .06$, CFI/TLI $> .95$ using the scaled versions for WLSMV (For reference only, as the correct approach would be to use the cutoffs derived from the DFI); (3) **Local fit** — standardized residuals $> |2|$, and modification indices (MI) > 3.84; (4) **Reliability** — GLB and composite reliability (omega) should exceed .70 for adequate internal consistency.

Table 3: Fit indices for the 4-factor CFA model

^{2}	df	p	CFI	TLI	RMSEA	RMSEA 90% CI	SRMR
1732.82	245	0	0.941	0.934	0.076	[0.073, 0.08]	0.057

Table 4: Standardized factor loadings for the 4-factor model

Factor	Item	Std.Loading	SE	p
psycho	Q5_P	0.740	0.017	$< .001$
psycho	Q6_P	0.670	0.018	$< .001$
psycho	Q7_P	0.664	0.018	$< .001$
psycho	Q11_P	0.659	0.020	$< .001$
psycho	Q19_P	0.860	0.012	$< .001$
psycho	Q26_P	0.640	0.020	$< .001$
physical	Q3_F	0.508	0.026	$< .001$
physical	Q4_F	0.423	0.029	$< .001$
physical	Q10_F	0.817	0.012	$< .001$
physical	Q15_F	0.639	0.025	$< .001$
physical	Q16_F	0.593	0.023	$< .001$
physical	Q17_F	0.921	0.007	$< .001$
physical	Q18_F	0.886	0.009	$< .001$
social	Q20_S	0.818	0.018	$< .001$

Table 4: Standardized factor loadings for the 4-factor model

Factor	Item	Std.Loading	SE	p
social	Q21_S	0.583	0.027	$< .001$
social	Q22_S	0.751	0.020	$< .001$
environment	Q8_A	0.510	0.028	$< .001$
environment	Q9_A	0.624	0.024	$< .001$
environment	Q12_A	0.712	0.021	$< .001$
environment	Q13_A	0.649	0.023	$< .001$
environment	Q14_A	0.766	0.020	$< .001$
environment	Q23_A	0.571	0.027	$< .001$
environment	Q24_A	0.482	0.028	$< .001$
environment	Q25_A	0.529	0.028	$< .001$

Table 5: Factor correlations for the 4-factor model

Factor 1	Factor 2	r	SE	p
psycho	physical	0.909	0.010	$< .001$
psycho	social	0.771	0.019	$< .001$
psycho	environment	0.725	0.021	$< .001$
physical	social	0.638	0.024	$< .001$
physical	environment	0.630	0.022	$< .001$
social	environment	0.550	0.028	$< .001$

Examining the output, the scaled chi-square test is statistically significant (expected for large N), but this alone is not informative for model evaluation. The scaled fit indices reveal a mediocre fit: the CFI and TLI values fall slightly below the conventional .95 threshold, and the RMSEA exceeds .06. The SRMR is within acceptable range. This pattern is consistent with a Level 2 misspecification according to DFI standards — the model captures the general structure but has notable local misfits.

Regarding factor loadings, most items show adequate standardized loadings (Std.all $>$ .50), indicating substantial association with their respective factors. However, Q4 (Physical) shows a relatively lower loading, and the R^2 values for Q4 (Physical) and Q24 (Environment) are notably low, suggesting these items are not well-explained by their assigned factor.

5.3.1 Modification Indices

Modification indices (MI) estimate the expected decrease in the chi-square statistic if a currently fixed parameter were freed. An MI $>$ 3.84 is statistically significant at = .05, and practically relevant MIs typically exceed 10. However, not all high MIs should be acted upon — only those with theoretical justification.

Table 6: Top 20 modification indices for the 4-factor model

	lhs	op	rhs	mi	epc
289	Q5_P	~~	Q14_A	177.218	0.278
254	environment	=~	Q5_P	175.788	0.503
544	Q24_A	~~	Q25_A	124.552	0.282
216	physical	=~	Q5_P	119.025	-1.039
467	Q17_F	~~	Q18_F	92.645	0.162
263	environment	=~	Q15_F	89.086	0.347
214	psycho	=~	Q24_A	80.033	-0.333
231	physical	=~	Q24_A	78.731	-0.289
200	psycho	=~	Q10_F	55.182	0.631
265	environment	=~	Q17_F	48.628	-0.251
266	environment	=~	Q18_F	44.622	-0.236
280	Q5_P	~~	Q17_F	44.099	-0.193
281	Q5_P	~~	Q18_F	41.613	-0.188
258	environment	=~	Q19_P	37.766	-0.254
428	Q10_F	~~	Q18_F	34.964	-0.123
252	social	=~	Q24_A	33.765	-0.206
203	psycho	=~	Q17_F	33.378	-0.549
394	Q3_F	~~	Q15_F	32.487	0.181
212	psycho	=~	Q14_A	31.896	0.221
223	physical	=~	Q21_S	30.562	0.233

The modification indices reveal a critical finding: **item Q5 appears in multiple high-MI entries**, suggesting cross-loadings on factors other than Psychological. This is a strong empirical signal that Q5 may not function as intended within the four-factor structure. Additionally, several MIs suggest correlated residuals between items across domains, reflecting the high correlation between Physical and Psychological factors.

⚠ Red Flag: Item Q5

When a single item appears repeatedly among the highest modification indices, this suggests a systematic misfit rather than isolated local strain. Freeing individual parameters (e.g., adding cross-loadings) is generally not recommended as a post hoc strategy — it risks capitalizing on sample-specific patterns. The better approach is to evaluate whether the item should be removed from the model.

5.3.2 Residual Correlations

Standardized residuals greater than |2| indicate item pairs whose empirical correlation is poorly reproduced by the model. The correlation residuals below show the discrepancies between model-implied and observed correlations.

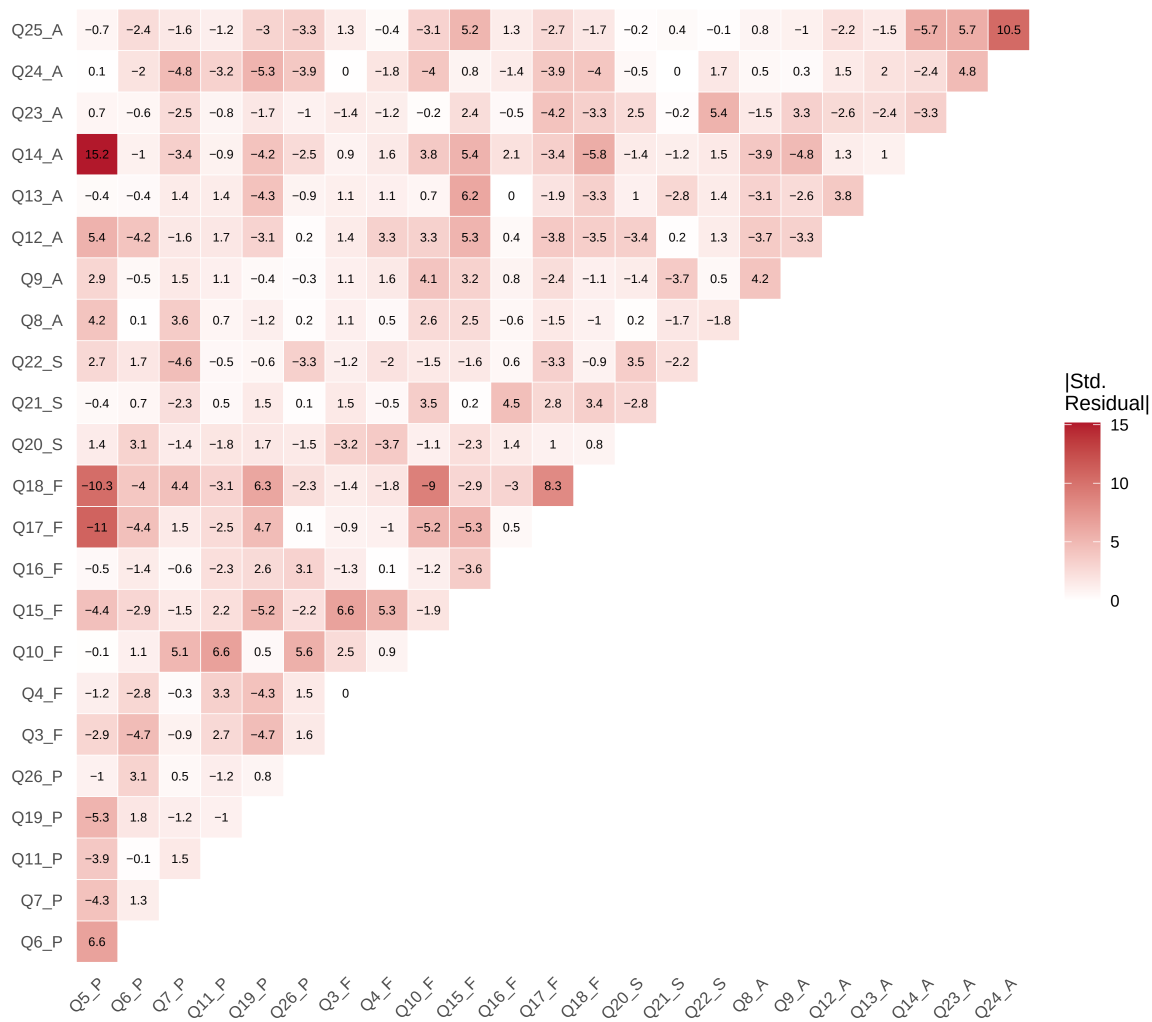


Figure 1: Heatmap of standardized residual correlations for the 4-factor model

Figure 1 provides a global view of local misfit. Color intensity reflects the absolute magnitude of standardized residuals — darker red indicates poorer reproduction of that pairwise correlation by the model, regardless of direction. Cell labels retain the sign: positive values indicate the model underestimates the correlation, negative values indicate overestimation. Pairs exceeding |2| deserve closer inspection.

The residual analysis confirms the patterns identified by the modification indices: the largest standardized residuals involve Q5 and items from other domains, reinforcing the conclusion that this item introduces systematic misfit.

5.3.3 Reliability Coefficients

We compute reliability using `semTools::reliability()`, which provides: alpha (Cronbach's), omega (composite reliability, preferred for CFA), omega2 (Bentler's), omega3 (McDonald's), and AVE (average variance extracted). AVE $> .50$ indicates that the factor captures more variance from its items than is due to measurement error.

Table 7: Reliability coefficients for the 4-factor model

	psycho	physical	social	environment
alpha	0.820	0.823	0.687	0.787
alpha.ord	0.854	0.863	0.741	0.820
omega	0.821	0.801	0.707	0.794
omega2	0.821	0.801	0.707	0.794
omega3	0.823	0.811	0.723	0.797
avevar	0.504	0.500	0.524	0.375

All four domains show composite reliability (omega) above .70, indicating adequate internal consistency. The AVE values are acceptable for most domains, with the Environment domain showing a slightly lower value — consistent with its broader and more heterogeneous item content. The Social domain, despite having only three items, shows strong reliability due to the high factor loadings of Q20, Q21, and Q22.

5.3.4 Preliminary Evaluation Summary

Based on the overall and local fit analysis:

- **Overall fit is mediocre**, compatible with a Level 2 misspecification (DFI)
- **Local indices indicate**: (a) high correlation between Physical and Psychological factors (suggesting potential hierarchical structure);

  (b) low factor loadings for Q4 and Q24; (c) **item Q5 is systematically problematic** — it appears in many high MIs and contributes to local misfit

- **Reliability is adequate** for all domains

5.4 Path Diagrams

The path diagram below visualize the estimated model. Latent factors are shown as ovals, observed items as rectangles, and standardized factor loadings as edge labels. The circle layout better displays interfactor correlations.

> **Layout Note**
>
> Consider this layout for educational purposes, as for your article you should create something better formatted, which will almost always be done by hand.

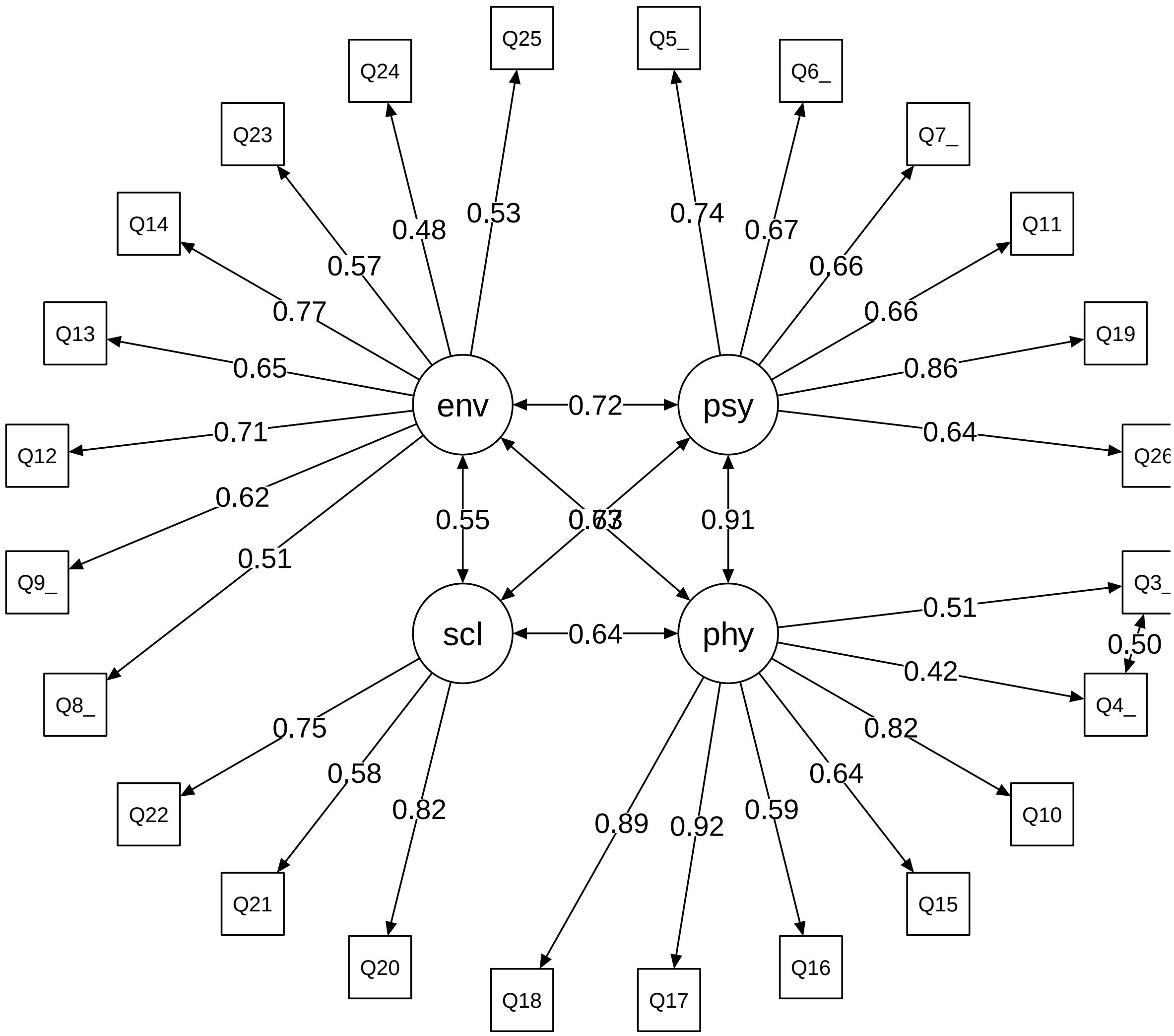


Figure 2: Path diagram of the 4-factor CFA model (circle layout)

The circle layout in Figure 2 reveals the strong correlations among all four factors, particularly between Psychological and Physical domains. This pattern motivates the exploration of bifactor and second-order models in the following sections.

6 4-Factor Bifactor Model

Given the high interfactor correlations observed in the 4-factor model, a bifactor structure is a natural alternative. In a bifactor model, a general factor (here, overall Quality of Life — QOL) loads directly on all items alongside specific group factors for each domain. The critical assumption is that all factors — general and specific — are **orthogonal** (uncorrelated).

This model tests whether there is a meaningful general QOL construct underlying all items, and how much unique variance remains in each specific domain after accounting for the general factor. Item Q5 is excluded from this model due to convergence problems, which further supports the preliminary evidence that Q5 is problematic.

> **On Bifactor Models**
>
> Bifactor models have gained popularity in psychometrics because they often provide better fit indices than correlated-factor models. However, as discussed in Rogers (2024), some authors argue that this improved fit is partly an artifact of the greater number of parameters, and that the orthogonality assumption rarely holds in practice. These considerations will be important when comparing models.

6.1 Model Specification

In bifactor model syntax, the **general factor must be specified first**. The Q3–Q4 residual correlation is no longer needed because both items now load on the general factor, which absorbs their shared variance.

6.2 Model Estimation

The `orthogonal = TRUE` argument is **required** for bifactor models, enforcing zero correlations between all factors.

6.3 Model Evaluation

Table 8: Fit indices for the bifactor 4-factor CFA model

^{2}	df	p	CFI	TLI	RMSEA	RMSEA 90% CI	SRMR
1212.68	207	0	0.958	0.949	0.068	[0.064, 0.072]	0.049

Table 9: Standardized factor loadings for the bifactor 4-factor model

Factor	Item	Std.Loading	SE	p
QOL	Q6_P	0.632	0.022	$<$ .001
QOL	Q7_P	0.668	0.019	$<$ .001
QOL	Q11_P	0.666	0.020	$<$ .001
QOL	Q19_P	0.851	0.012	$<$ .001
QOL	Q26_P	0.634	0.022	$<$ .001
QOL	Q3_F	0.422	0.030	$<$ .001
QOL	Q4_F	0.349	0.031	$<$ .001
QOL	Q10_F	0.785	0.014	$<$ .001
QOL	Q15_F	0.584	0.027	$<$ .001
QOL	Q16_F	0.573	0.023	$<$ .001
QOL	Q17_F	0.875	0.012	$<$ .001
QOL	Q18_F	0.842	0.013	$<$ .001
QOL	Q20_S	0.598	0.021	$<$ .001
QOL	Q21_S	0.452	0.026	$<$ .001
QOL	Q22_S	0.541	0.024	$<$ .001
QOL	Q8_A	0.372	0.028	$<$ .001
QOL	Q9_A	0.454	0.026	$<$ .001
QOL	Q12_A	0.499	0.024	$<$ .001
QOL	Q13_A	0.461	0.025	$<$ .001
QOL	Q14_A	0.527	0.024	$<$ .001
QOL	Q23_A	0.380	0.029	$<$ .001
QOL	Q24_A	0.246	0.031	$<$ .001
QOL	Q25_A	0.328	0.029	$<$ .001
psycho	Q6_P	0.696	0.479	0.146
psycho	Q7_P	0.059	0.051	0.247
psycho	Q11_P	0.021	0.032	0.502
psycho	Q19_P	0.085	0.066	0.2
psycho	Q26_P	0.135	0.102	0.184
physical	Q3_F	0.690	0.042	$<$ .001
physical	Q4_F	0.606	0.039	$<$ .001
physical	Q10_F	0.116	0.030	$<$ .001
physical	Q15_F	0.290	0.037	$<$ .001
physical	Q16_F	0.059	0.037	0.107
physical	Q17_F	0.268	0.031	$<$ .001
physical	Q18_F	0.257	0.031	$<$ .001
social	Q20_S	0.566	0.052	$<$ .001
social	Q21_S	0.269	0.035	$<$ .001
social	Q22_S	0.567	0.052	$<$ .001
environment	Q8_A	0.300	0.032	$<$ .001

Table 9: Standardized factor loadings for the bifactor 4-factor model

Factor	Item	Std.Loading	SE	p
environment	Q9_A	0.386	0.029	$<$ .001
environment	Q12_A	0.485	0.026	$<$ .001
environment	Q13_A	0.453	0.028	$<$ .001
environment	Q14_A	0.441	0.028	$<$ .001
environment	Q23_A	0.468	0.029	$<$ .001
environment	Q24_A	0.603	0.028	$<$ .001
environment	Q25_A	0.518	0.027	$<$ .001

The bifactor model shows improved fit compared to the original 4-factor model, with slightly better CFI, TLI, and RMSEA values. However, the improvement is modest and the fit remains in the mediocre range (Level 2 of DFI).

Examining the standardized loadings, the general QOL factor captures substantial variance from most items. Notably, the **Psychological specific factor shows very low loadings** — most of the variance that was previously attributed to the Psychological domain is now absorbed by the general factor. This pattern suggests that psychological well-being items are primarily indicators of overall QOL rather than a distinct domain.

Table 10: Top 20 modification indices for the bifactor model

	lhs	op	rhs	mi	epc
462	Q17_F	~~	Q18_F	155.946	0.215
280	environment	=~	Q15_F	114.813	0.300
387	Q3_F	~~	Q4_F	58.434	0.819
283	environment	=~	Q18_F	46.170	-0.177
286	environment	=~	Q22_S	45.651	0.170
539	Q24_A	~~	Q25_A	44.606	0.231
282	environment	=~	Q17_F	42.692	-0.170
166	social	~~	environment	38.239	0.196
269	social	=~	Q23_A	36.821	0.232
164	physical	~~	social	34.825	-0.274
162	psycho	~~	social	33.459	0.280
252	social	=~	Q6_P	31.214	0.184
526	Q12_A	~~	Q14_A	31.078	0.152
509	Q22_S	~~	Q23_A	30.368	0.165
512	Q8_A	~~	Q9_A	29.681	0.156
391	Q3_F	~~	Q17_F	29.502	-0.244
392	Q3_F	~~	Q18_F	28.622	-0.239
448	Q15_F	~~	Q25_A	26.451	0.154

Table 10: Top 20 modification indices for the bifactor model

	lhs	op	rhs	mi	epc
241	physical	=~	Q20_S	23.444	-0.174
408	Q4_F	~~	Q18_F	23.223	-0.198

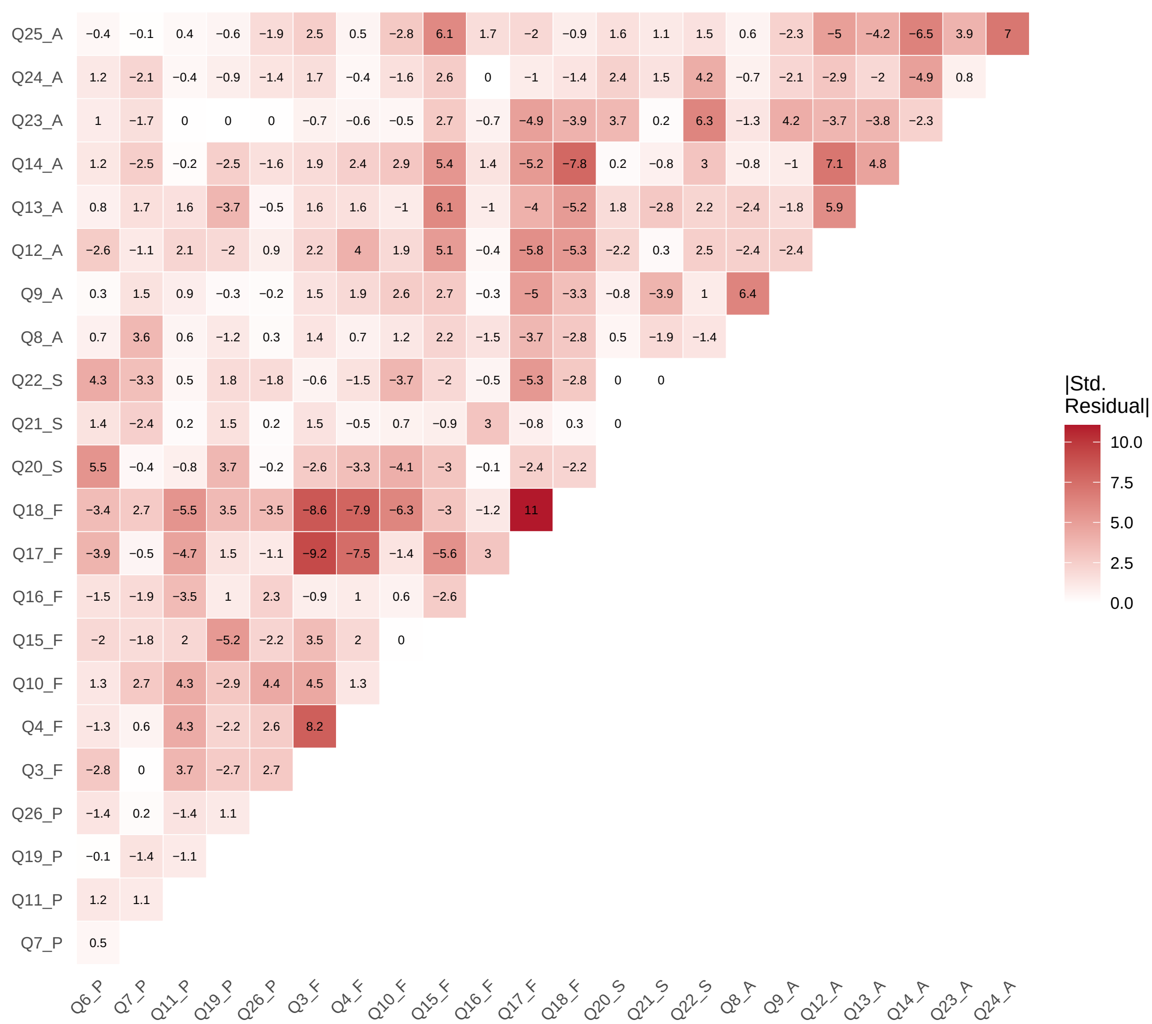


Figure 3: Heatmap of standardized residual correlations for the bifactor 4-factor model

6.3.1 Reliability

For bifactor models, omega from `psych::omegaFromSem()` provides the most appropriate reliability decomposition: **omega hierarchical (H)** estimates the proportion of total score variance attributable to the general factor, while **omega subscale (S)** estimates the proportion attributable to each specific factor after controlling for the general factor.

Table 11: Omega Coefficients - Bifactor Model

Dimension	T	H	S
General Factor	0.942	0.837	0.111
Psychological	0.864	0.797	0.067
Physical	0.926	0.731	0.195
Social	0.758	0.427	0.331
Environmental	0.847	0.376	0.471

The omega hierarchical value indicates that the general QOL factor accounts for a substantial proportion of reliable variance. The Psychological specific factor contributes very little unique reliable variance beyond the general factor, confirming the pattern observed in the loadings. The Physical, Social, and Environment specific factors retain more meaningful unique variance.

6.3.2 Preliminary Evaluation Summary

- **Overall fit has improved** but remains mediocre (Level 2 of DFI, although not directly comparable)
- The **general (G) factor is statistically relevant** and explains substantial variance across all items
- The **Psychological specific factor loses most of its importance** — its variance is almost entirely captured by the general QOL factor
- The orthogonality assumption required for this model is a strong restriction that may not hold empirically

6.4 Path Diagram

The `tree3` layout is specifically designed for bifactor diagrams, placing the general factor centrally and specific factors on the sides.

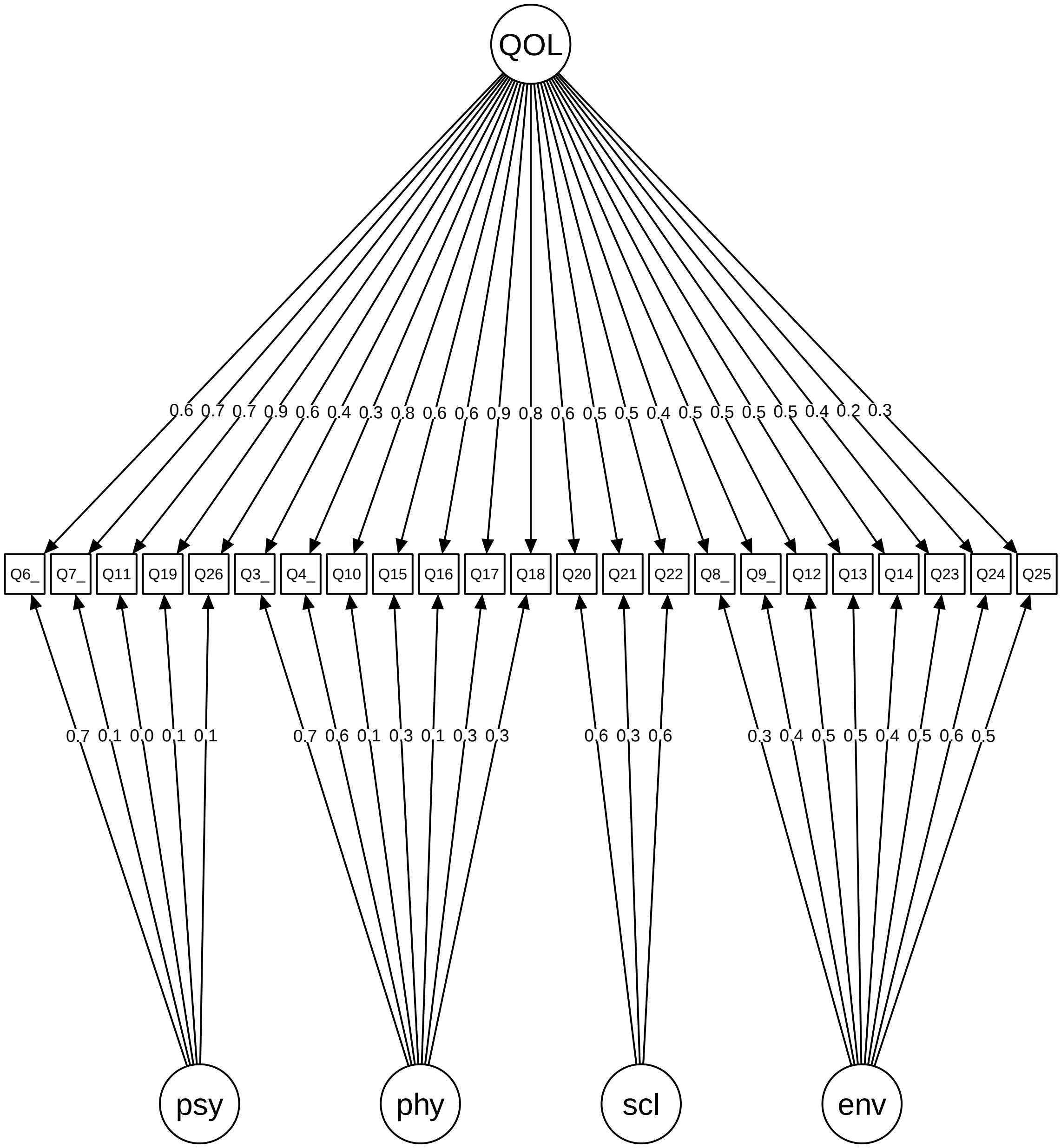


Figure 4: Path diagram of the bifactor CFA model

Path Diagram Layout

You can test other formatting parameters (edge.label.cex, label.cex, edge.label.position, etc) to simulate path diagrams with better layouts.

7 Second-Order CFA Model

An alternative to the bifactor model is the second-order (hierarchical) model. Here, the general QOL factor does **not** load directly on items. Instead, it loads on the four first-order factors, which in turn load on the observed items. This structure implies that the influence of the general factor on individual items is *mediated* by the domain-specific factors — a conceptually different interpretation from the bifactor model.

7.1 Model Specification

Item Q5 is excluded for comparability with the bifactor model. A technical requirement is that the Psychological factor variance must be fixed to zero for model convergence (empirical underidentification), suggesting that essentially all Psychological domain variance is explained by the higher-order QOL factor.

Convergence Note

Fixing the Psychological factor variance to zero (`psycho ~~ 0*psycho`) is necessary for model convergence. This is not merely a technical trick — it has substantive meaning: nearly all variance in the Psychological domain is captured by the general QOL factor. This finding is consistent with the bifactor results, where the Psychological specific factor had minimal importance.

7.2 Model Estimation

Note that `std.lv` is not used here because the second-order structure imposes its own identification constraints through the higher-order factor loadings.

7.3 Model Evaluation

The second-order model fit is comparable to the other models, though the convergence issue with the Psychological domain is a notable limitation. The second-order loadings (QOL → first-order factors) reveal the relative contribution of each domain to overall quality of life.

Table 12: Fit indices for the 2nd order 4-factor CFA model

^{2}	df	p	CFI	TLI	RMSEA	RMSEA 90% CI	SRMR
1285.17	226	0	0.956	0.951	0.067	[0.063, 0.071]	0.054

Table 13: Standardized factor loadings for the 2nd order 4-factor model

Factor	Item	Std.Loading	SE	p
psycho	Q6_P	0.660	0.019	$< .001$
psycho	Q7_P	0.678	0.018	$< .001$
psycho	Q11_P	0.671	0.020	$< .001$
psycho	Q19_P	0.872	0.012	$< .001$
psycho	Q26_P	0.648	0.020	$< .001$
physical	Q3_F	0.509	0.026	$< .001$
physical	Q4_F	0.422	0.029	$< .001$
physical	Q10_F	0.811	0.012	$< .001$
physical	Q15_F	0.642	0.025	$< .001$
physical	Q16_F	0.590	0.023	$< .001$
physical	Q17_F	0.923	0.007	$< .001$
physical	Q18_F	0.888	0.009	$< .001$
social	Q20_S	0.818	0.019	$< .001$
social	Q21_S	0.588	0.027	$< .001$
social	Q22_S	0.747	0.020	$< .001$
environment	Q8_A	0.510	0.028	$< .001$
environment	Q9_A	0.630	0.025	$< .001$
environment	Q12_A	0.715	0.021	$< .001$
environment	Q13_A	0.663	0.023	$< .001$
environment	Q14_A	0.731	0.021	$< .001$
environment	Q23_A	0.579	0.027	$< .001$
environment	Q24_A	0.492	0.028	$< .001$
environment	Q25_A	0.541	0.027	$< .001$
QOL	psycho	1.000	0.000	NA
QOL	physical	0.912	0.010	$< .001$
QOL	social	0.740	0.020	$< .001$
QOL	environment	0.685	0.021	$< .001$

Table 14: Top 20 modification indices for the 2nd order model

	lhs	op	rhs	mi	epc
533	Q24_A	~~	Q25_A	115.469	0.274
252	environment	=~	Q15_F	94.778	0.673
456	Q17_F	~~	Q18_F	82.991	0.156
206	psycho	=~	Q24_A	74.190	-0.449
280	QOL	=~	Q24_A	74.190	-0.449
222	physical	=~	Q24_A	72.314	-0.523
192	psycho	=~	Q10_F	54.814	1.152
266	QOL	=~	Q10_F	54.814	1.152
255	environment	=~	Q18_F	36.927	-0.394
254	environment	=~	Q17_F	36.682	-0.395
417	Q10_F	~~	Q18_F	32.049	-0.119
383	Q3_F	~~	Q15_F	31.584	0.179
195	psycho	=~	Q17_F	28.227	-0.929
269	QOL	=~	Q17_F	28.227	-0.929
242	social	=~	Q24_A	27.741	-0.206
503	Q22_S	~~	Q23_A	24.588	0.147
532	Q23_A	~~	Q25_A	24.194	0.141
208	physical	=~	Q6_P	22.864	-1.054
215	physical	=~	Q22_S	22.662	-0.599
438	Q15_F	~~	Q13_A	22.272	0.148

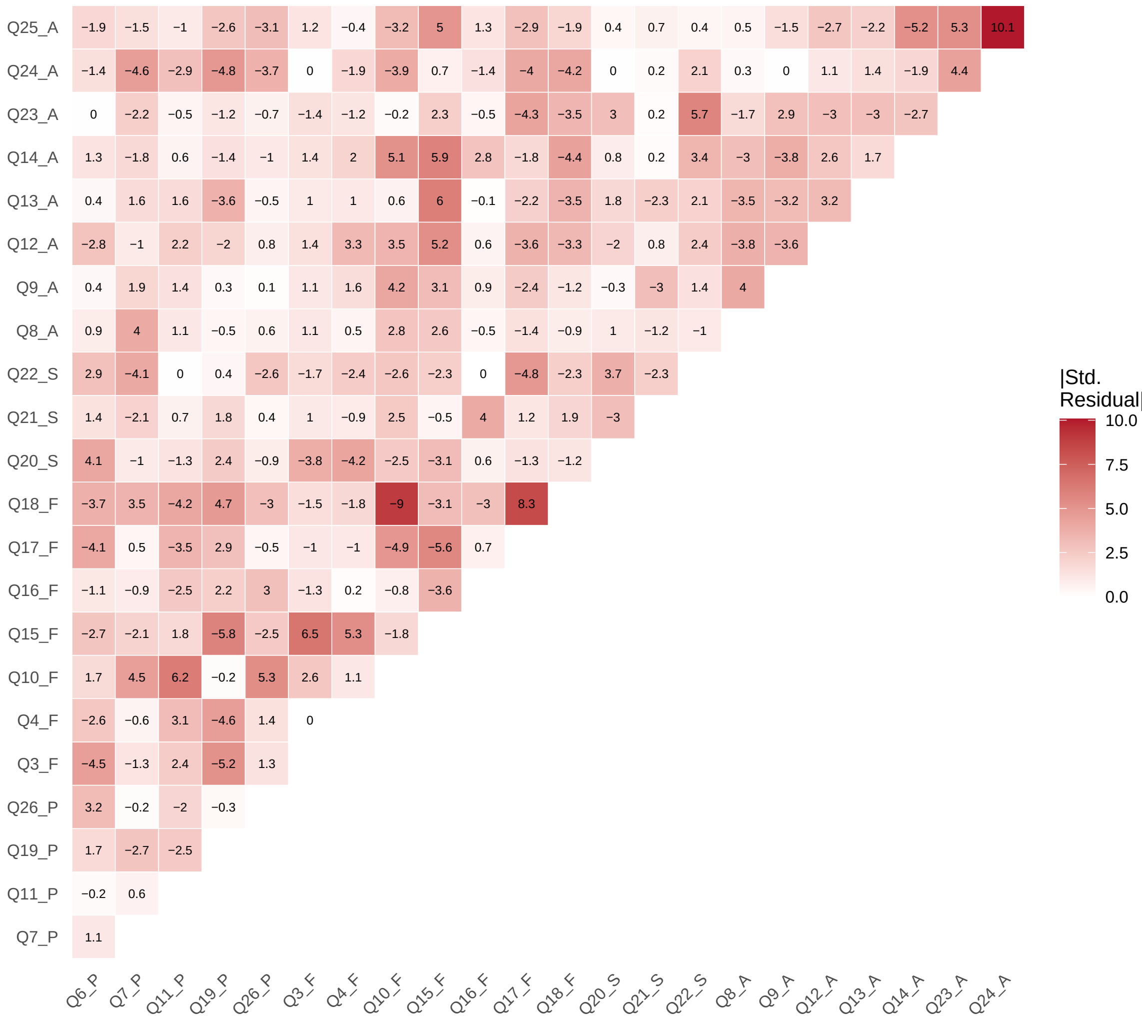


Figure 5: Heatmap of standardized residual correlations for the 2nd order 4-factor model

7.3.1 Reliability

For the first-order factors, we use `semTools::reliability()`. For the second-order QOL factor, `semTools::reliabilityL2()` provides the appropriate decomposition.

Table 15: Reliability coefficients for first-order factors (second-order model)

	psycho	physical	social	environment
alpha	0.793	0.823	0.687	0.787
alpha.ord	0.831	0.863	0.741	0.820

Table 15: Reliability coefficients for first-order factors (second-order model)

	psycho	physical	social	environment
omega	0.794	0.801	0.708	0.795
omega2	0.794	0.801	0.708	0.795
omega3	0.794	0.810	0.725	0.802
avevar	0.505	0.499	0.525	0.377

Table 16: Second-order factor (QOL) reliability

omegaL1	omegaL2	partialOmegaL1
0.838	0.899	0.926

The second-order QOL factor shows adequate reliability, indicating that the four first-order domains collectively serve as reliable indicators of overall quality of life. First-order reliability values remain consistent with those observed in the standard 4-factor model.

7.4 Path Diagram

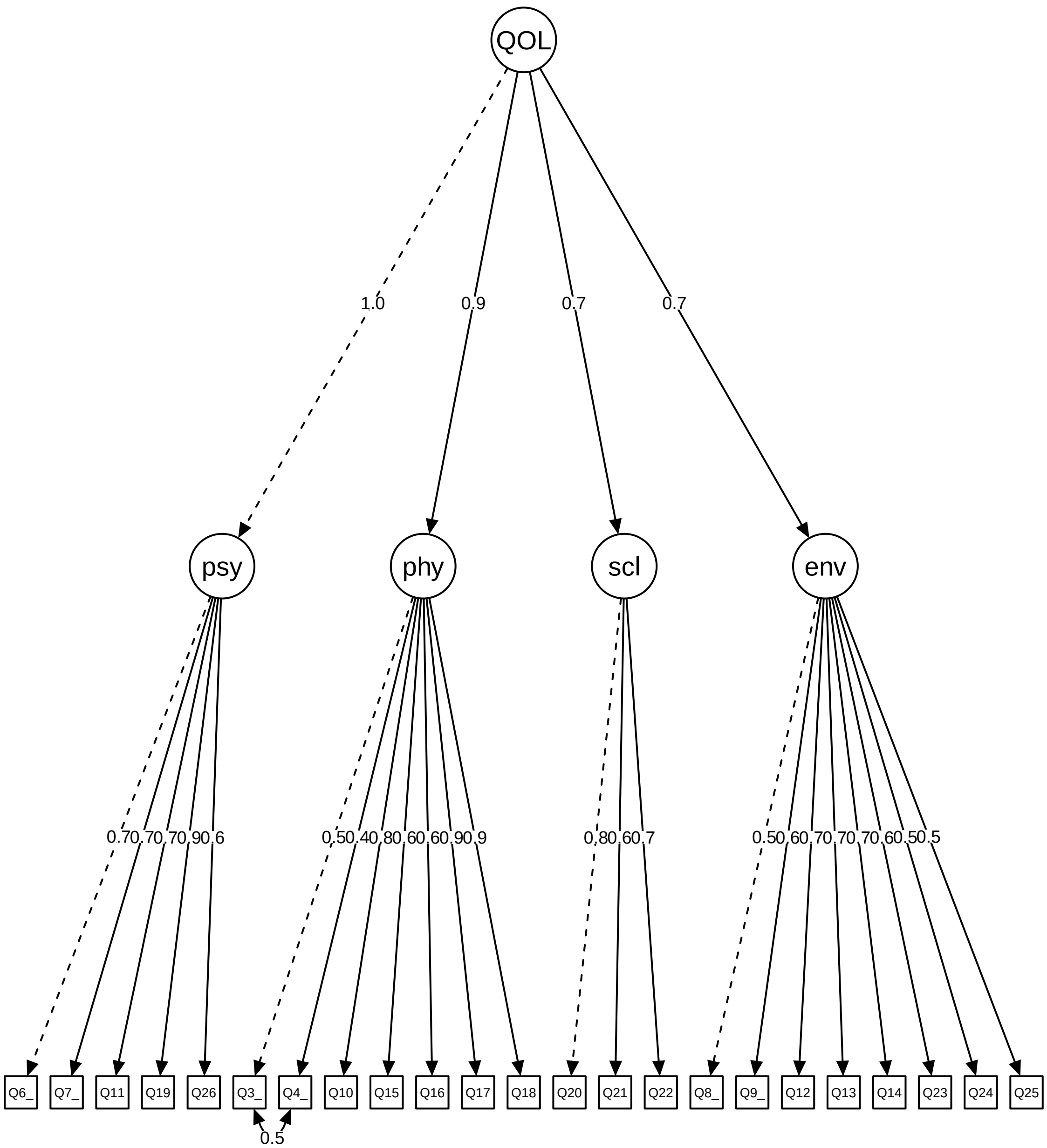


Figure 6: Path diagram of the second-order CFA model

8 4-Factor Model Without Item Q5

The analyses above converged on a consistent finding: **item Q5 is problematic**. It appeared in multiple high modification indices in the original 4-factor model, caused convergence problems in the bifactor model, and its removal was necessary for comparability across alternative models. This is a common scenario in applied CFA — items that appear sound from a content validity perspective may not perform well empirically in a given sample or cultural context.

8.1 Model Specification

This section presents the revised 4-factor model excluding Q5, retaining the correlated residuals between Q3 and Q4. This model will serve as the basis for the final model comparison and post hoc power analysis.

8.2 Model Estimation

8.3 Model Evaluation

Table 17: Fit indices for the 4-factor CFA model (Q5 excluded)

^{2}	df	p	CFI	TLI	RMSEA	RMSEA 90% CI	SRMR
1285.46	223	0	0.956	0.95	0.067	[0.064, 0.071]	0.053

Table 18: Standardized factor loadings for the 4-factor model (Q5 excluded)

Factor	Item	Std.Loading	SE	p
psycho	Q6_P	0.659	0.019	$< .001$
psycho	Q7_P	0.677	0.018	$< .001$
psycho	Q11_P	0.671	0.019	$< .001$
psycho	Q19_P	0.869	0.012	$< .001$
psycho	Q26_P	0.648	0.020	$< .001$
physical	Q3_F	0.510	0.026	$< .001$
physical	Q4_F	0.423	0.029	$< .001$
physical	Q10_F	0.811	0.012	$< .001$
physical	Q15_F	0.643	0.025	$< .001$
physical	Q16_F	0.590	0.023	$< .001$
physical	Q17_F	0.923	0.007	$< .001$
physical	Q18_F	0.888	0.009	$< .001$
social	Q20_S	0.819	0.019	$< .001$

Table 18: Standardized factor loadings for the 4-factor model (Q5 excluded)

Factor	Item	Std.Loading	SE	p
social	Q21_S	0.586	0.027	$< .001$
social	Q22_S	0.748	0.020	$< .001$
environment	Q8_A	0.510	0.028	$< .001$
environment	Q9_A	0.629	0.025	$< .001$
environment	Q12_A	0.715	0.021	$< .001$
environment	Q13_A	0.663	0.023	$< .001$
environment	Q14_A	0.732	0.021	$< .001$
environment	Q23_A	0.580	0.027	$< .001$
environment	Q24_A	0.492	0.028	$< .001$
environment	Q25_A	0.542	0.027	$< .001$

Table 19: Factor correlations for the 4-factor model (Q5 excluded)

Factor 1	Factor 2	r	SE	p
psycho	physical	0.922	0.010	$< .001$
psycho	social	0.755	0.019	$< .001$
psycho	environment	0.659	0.024	$< .001$
physical	social	0.638	0.024	$< .001$
physical	environment	0.630	0.022	$< .001$
social	environment	0.550	0.028	$< .001$

After removing Q5, the overall fit shows improvement. All remaining items exhibit adequate standardized loadings, and the model converges without issues. The Q5 removal resolves the most prominent source of local misfit identified in the original 4-factor model.

Table 20: Top 20 modification indices for the 4-factor model (Q5 excluded)

	lhs	op	rhs	mi	epc
510	Q24_A	~~	Q25_A	114.934	0.273
252	environment	=~	Q15_F	94.407	0.351
433	Q17_F	~~	Q18_F	84.659	0.157
222	physical	=~	Q24_A	78.505	-0.281
206	psycho	=~	Q24_A	77.213	-0.289
254	environment	=~	Q17_F	43.449	-0.232
255	environment	=~	Q18_F	42.877	-0.228
192	psycho	=~	Q10_F	42.388	0.704
208	physical	=~	Q6_P	36.087	-0.700

Table 20: Top 20 modification indices for the 4-factor model (Q5 excluded)

	lhs	op	rhs	mi	epc
242	social	=~	Q24_A	33.243	-0.203
394	Q10_F	~~	Q18_F	31.926	-0.118
360	Q3_F	~~	Q15_F	31.211	0.178
214	physical	=~	Q21_S	31.008	0.235
198	psycho	=~	Q21_S	26.547	0.277
509	Q23_A	~~	Q25_A	23.889	0.140
415	Q15_F	~~	Q13_A	21.582	0.146
376	Q4_F	~~	Q15_F	21.431	0.147
416	Q15_F	~~	Q14_A	20.568	0.143
480	Q22_S	~~	Q23_A	18.885	0.131
224	social	=~	Q6_P	18.702	0.220

The modification indices are notably reduced compared to the original model. While some MIs remain above 3.84, they are smaller in magnitude and do not show the systematic pattern of misfit associated with Q5.

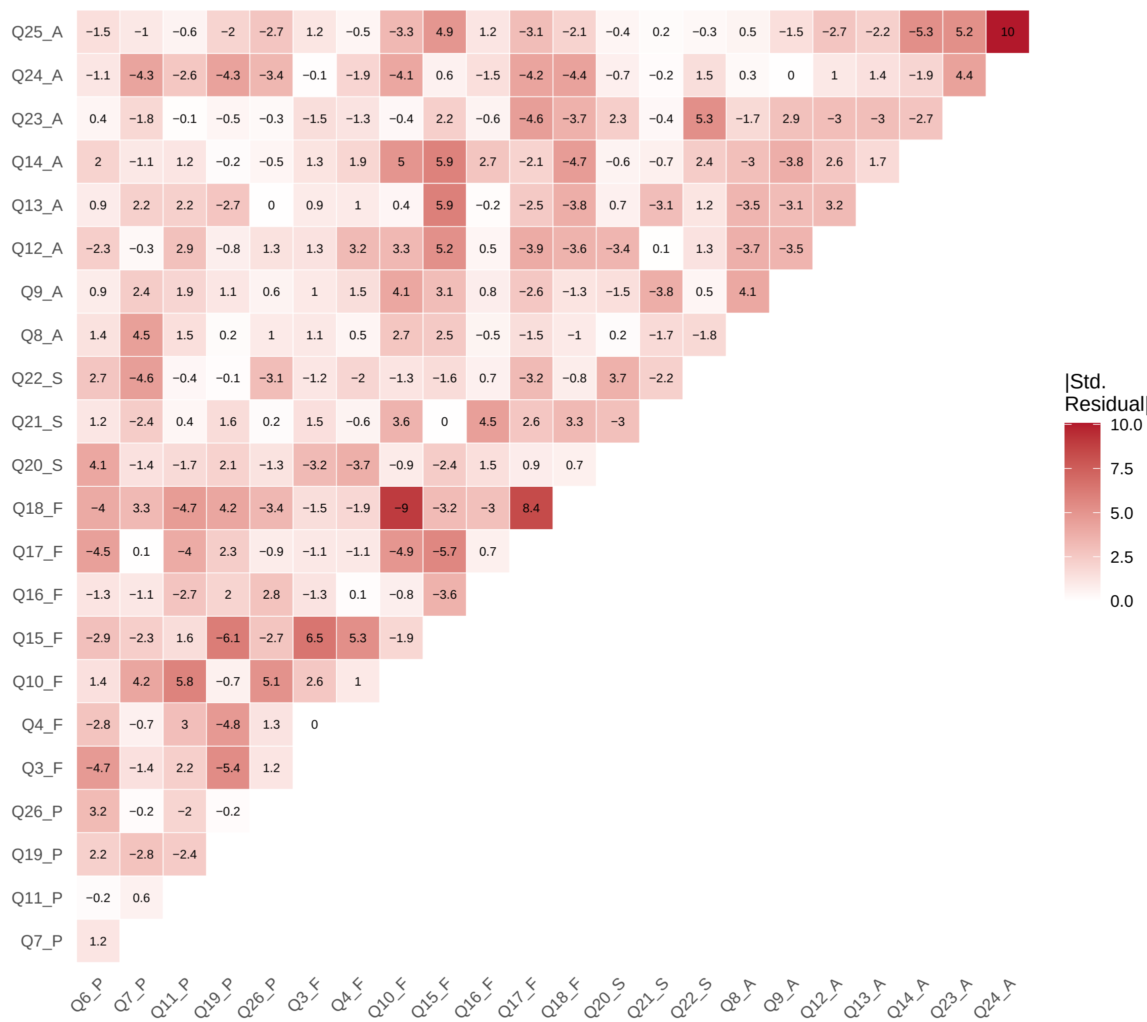


Figure 7: Heatmap of standardized residual correlations for the 4-factor model (Q5 excluded)

Table 21: Reliability coefficients for the revised 4-factor model

	psycho	physical	social	environment
alpha	0.793	0.823	0.687	0.787
alpha.ord	0.831	0.863	0.741	0.820
omega	0.793	0.801	0.708	0.795
omega2	0.793	0.801	0.708	0.795
omega3	0.792	0.811	0.725	0.802
avevar	0.504	0.500	0.524	0.377

Reliability for the Psychological domain remains adequate even after removing Q5, confirming that the item was not essential for capturing the construct. The remaining five items (Q6, Q7, Q11, Q19, Q26) provide a reliable measurement of psychological well-being. All other domains maintain their previous reliability levels.

8.4 Path Diagram

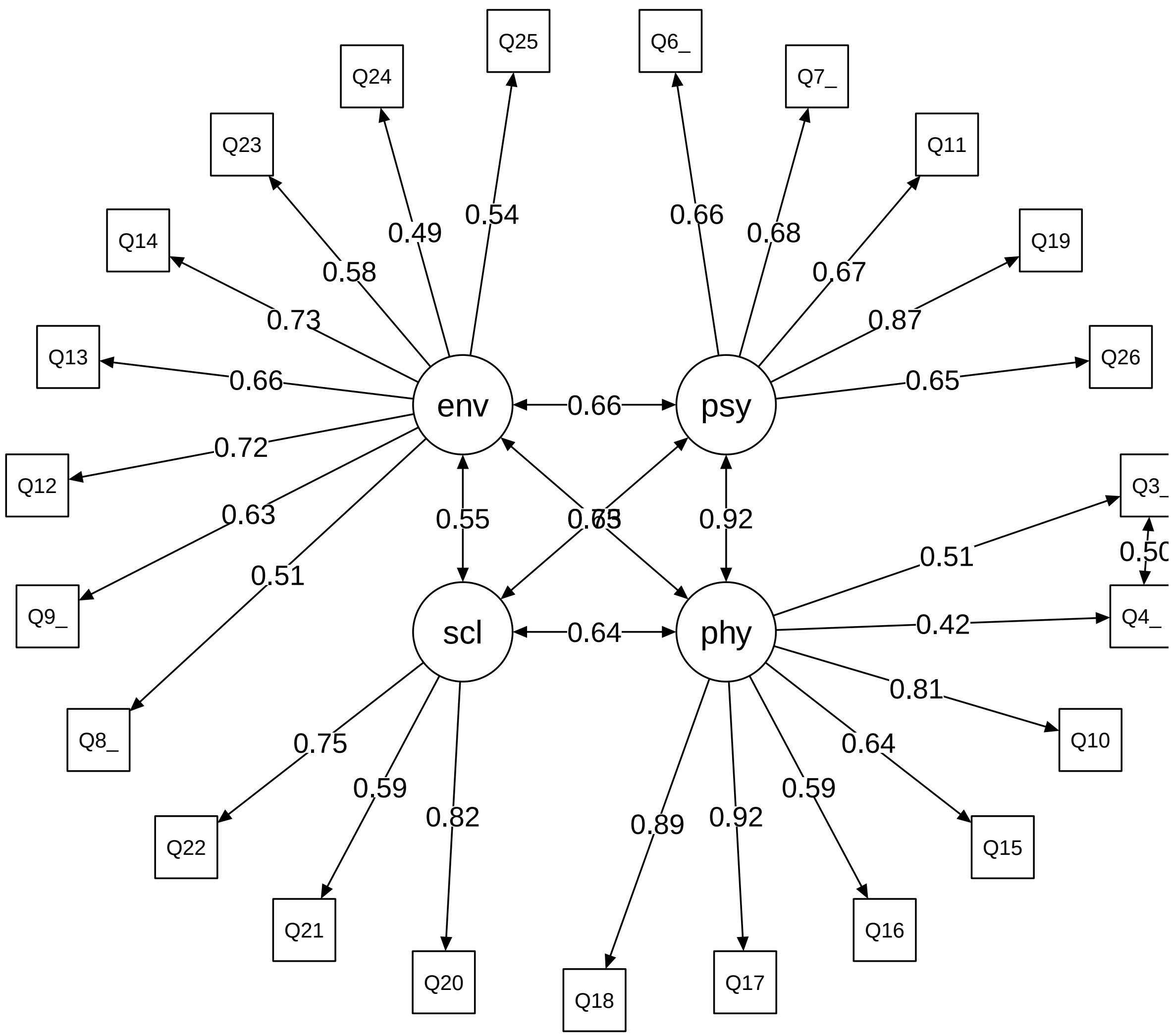


Figure 8: Path diagram of the revised 4-factor CFA model (without Q5)

9 Model Comparison

We now systematically compare the four models on key fit indices: SRMR, RMSEA (scaled), and CFI (scaled). These three indices capture complementary aspects of model fit — SRMR assesses the average discrepancy in correlations, RMSEA penalizes for model complexity, and CFI compares the model against a baseline independence model.

Table 22: Fit indices comparison across all CFA models

Model	SRMR	RMSEA	CFI
4-Factor (all items)	0.057	0.076	0.941
Bifactor	0.049	0.068	0.958
Second-Order	0.054	0.067	0.956
4-Factor (no Q5)	0.053	0.067	0.956

Table 22 shows that the four models exhibit a similar level of misspecification, although the original 4-factor model (with all items) performs slightly worse. The bifactor model shows a marginally better fit, which aligns with findings in the literature — this is likely due to the greater number of free parameters in bifactor structures, which mechanically tend to improve fit indices. As noted by Rogers (2024), some authors argue that bifactor models have been used to "salvage" traditional scales that are no longer sustainable under modern estimation and item selection techniques.

! Model Selection Recommendation

A known limitation of bifactor models is the assumption that specific factors are uncorrelated — an assumption that frequently does not hold in practice. Additionally, bifactor models can suffer from empirical underidentification and may capitalize on sample-specific variance.

The second-order model also showed convergence problems, requiring the Psychological factor variance to be fixed to zero.

Based on empirical evidence and practical considerations, the four correlated factor model excluding item Q5 is recommended for measuring WHOQOL-BREF in this sample. This model:

- Has acceptable overall and local fit
- Avoids the strong orthogonality assumption of bifactor models
- Is easier to interpret and communicate to non-technical audiences
- Has no convergence issues
- Maintains adequate reliability (omega $>$ .70) across all domains
- Is the most parsimonious adequate representation of the data

10 Post Hoc Power Analysis

The final step in our CFA workflow is a post hoc power analysis assessing whether the sample size (N = 1047) provides adequate statistical power for parameter estimation and misfit detection. We use Monte Carlo simulation via the `simsem` package: data are repeatedly generated from the fitted model and re-estimated, yielding empirical distributions of parameter estimates and fit indices.

> **Why Post Hoc Power Analysis?**
>
> While *a priori* power analysis is ideal for study planning, *post hoc* simulation-based power analysis provides essential information about: (1) **parameter stability** — whether estimates are consistent across replications; (2) **power to detect non-zero parameters** — whether loadings and correlations are reliably different from zero; (3) **coverage** — whether 95% confidence intervals perform as expected; and (4) **bias** — whether estimates and standard errors are unbiased.

We use the revised four-factor model (without Q5) as both the generating and analysis model. By generating data from the fitted lavaan object, we preserve the ordinal nature of the items and the estimated polychoric structure.

10.1 Define Cutoff Values

10.2 Run Simulation

> **Warning**
>
> To reduce computational time, we run 100 replications. For final published results, 1,000–5,000 replications are recommended for stable estimates. More replications yield narrower confidence intervals around power estimates but increase computation time proportionally.

10.3 Fit Index Cutoffs

Simulation-based cutoffs at = .05 provide model-specific and sample-specific alternatives to generic rules of thumb. These cutoffs represent the 95th (or 5th, for CFI) percentile of fit index distributions under the assumption that the model is correctly specified.

Table 23: Simulation-based fit index cutoffs at alpha = 0.05

	95%
srmr	0.0238
rmsea.scaled	0.0122
cfi.scaled	1.0000

10.4 Power to Detect Misfit

Table 24 shows the proportion of replications where fit indices exceeded rule-of-thumb cutoffs. Low values (near 0) indicate that the correctly specified model consistently shows good fit — which is the desired result.

Table 24: Proportion of replications indicating worse fit than rule-of-thumb cutoffs

cfi.scaled	rmsea.scaled	srmr
1	0	0

Interpreting Power to Detect Misfit

Low proportions in Table 24 mean that the model fits well in most replications — we have low power to reject it when it is correctly specified, which is exactly what we want. If these proportions were high, it would suggest that even when the model is correct, our sample size causes the fit indices to exceed cutoffs (i.e., the cutoffs are too strict for our model and N).

10.5 Sampling Distributions of Fit Indices

Figure 9 shows the empirical distributions of SRMR, RMSEA, and CFI across 100 replications, with rule-of-thumb cutoffs overlaid as vertical lines. The distributions should be concentrated well below (SRMR, RMSEA) or above (CFI) the cutoffs.

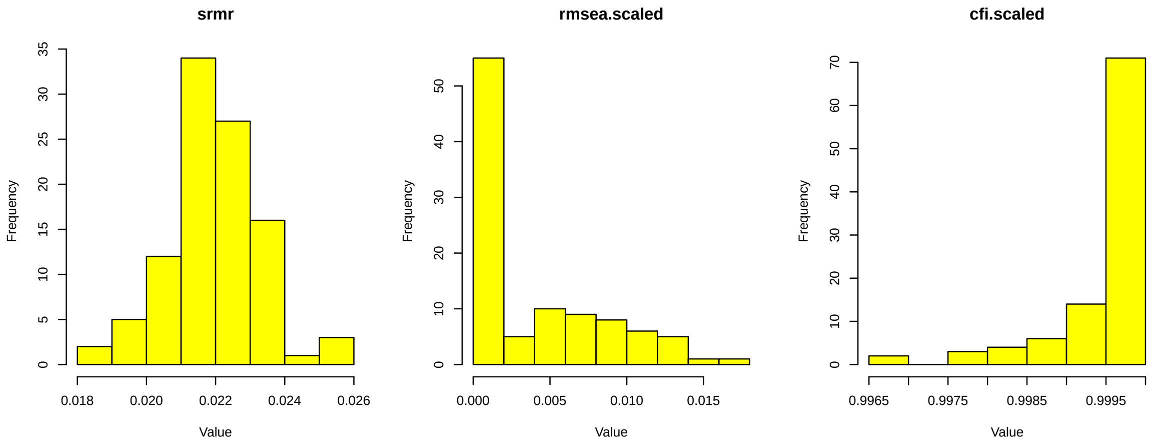


Figure 9: Sampling distributions of fit indices — Power of rejecting misspecified models

Figure 10 presents the same distributions with simulation-based cutoffs at = .05. The shaded areas represent the rejection regions.

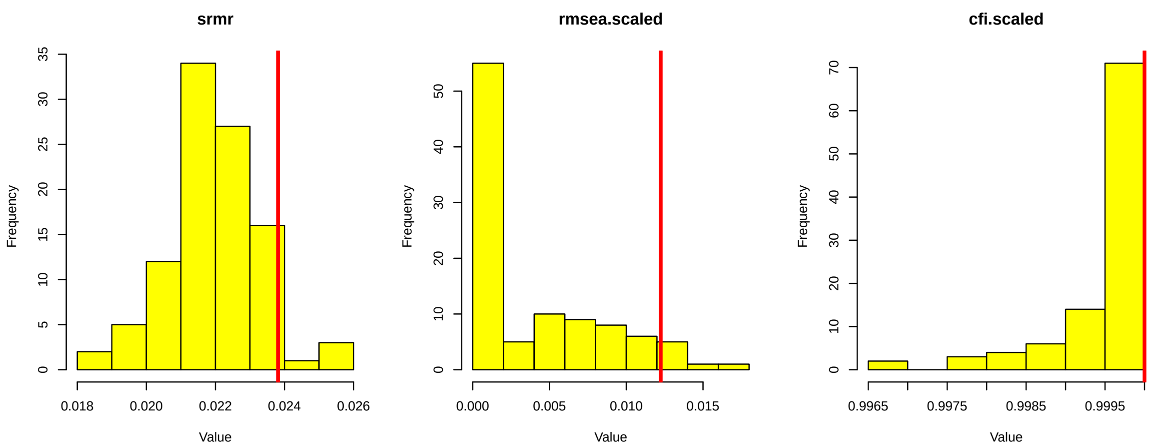


Figure 10: Sampling distributions of fit indices with simulation-based cutoffs at alpha = 0.05

10.6 Detailed Parameter Summary

The comprehensive parameter summary below reports, for each model parameter: coverage (proportion of 95% CIs containing the true value), power (proportion rejecting H : parameter = 0), relative bias in estimates, and relative bias in standard errors.

Table 25: Simulation Performance Statistics (n = 1,047, replications = 100)

Parameter	Power	Coverage	Bias %	SE Bias %
psycho=~Q6_P	1	0.96	0.004	0.060
psycho=~Q7_P	1	0.92	0.001	-0.067
psycho=~Q11_P	1	0.94	0.004	0.062
psycho=~Q19_P	1	0.93	0.005	-0.019
psycho=~Q26_P	1	0.96	0.006	0.125
physical=~Q3_F	1	0.98	0.002	-0.022
physical=~Q4_F	1	0.97	0.000	0.044
physical=~Q10_F	1	0.91	0.004	-0.066
physical=~Q15_F	1	0.92	-0.001	-0.076
physical=~Q16_F	1	0.92	0.003	-0.091
physical=~Q17_F	1	0.95	-0.001	0.011
physical=~Q18_F	1	0.93	0.001	-0.029
social=~Q20_S	1	0.94	0.005	0.029
social=~Q21_S	1	0.96	-0.006	0.070
social=~Q22_S	1	0.96	0.003	-0.032
environment=~Q8_A	1	0.95	-0.006	-0.026
environment=~Q9_A	1	0.98	-0.001	0.116
environment=~Q12_A	1	0.96	0.003	0.013
environment=~Q13_A	1	0.97	0.000	-0.011
environment=~Q14_A	1	0.93	-0.003	-0.040
environment=~Q23_A	1	0.97	-0.005	0.173
environment=~Q24_A	1	0.97	0.001	0.067
environment=~Q25_A	1	0.99	0.002	0.082
Q3_F~~Q4_F	1	0.97	-0.001	0.110
psycho~~physical	1	0.95	-0.004	-0.579
psycho~~social	1	0.95	-0.003	-0.093
psycho~~environment	1	0.94	0.005	0.056
physical~~social	1	0.98	-0.001	0.046
physical~~environment	1	0.88	0.009	-0.121
social~~environment	1	0.92	0.006	-0.124

Only few threshold parameters show power below .80 (see full report), and thresholds are generally not parameters of interest in CFA. All factor loadings and factor correlations are estimated with adequate power, indicating that the sample size is sufficient for reliable parameter estimation.

How to Read the Parameter Summary

Key benchmarks for evaluating simulation quality (Rogers, 2024):

- **Power $> .80$**: Adequate power to detect non-zero parameters
- **Coverage 0.91–0.98**: Confidence intervals perform as expected (neither too liberal nor too conservative)
- **Relative Bias $< |0.10|$**: Parameter estimates are unbiased (within 10% of true values)
- **Relative SE Bias $< |0.10|$**: Standard errors accurately estimate sampling variability

Parameters outside these ranges deserve attention. However, **thresholds are generally not parameters of interest** in CFA. The focus should be on factor loadings, factor correlations, and residual variances.

The simulation results confirm that the sample size of N = 1047 provides adequate power for estimating all parameters of substantive interest. Factor loadings and correlations are estimated with power $> .80$, coverage near .95, and negligible bias. This supports the reliability and generalizability of the CFA results presented throughout this tutorial.

11 Summary and Conclusions

This supplementary material demonstrated a complete CFA workflow for ordinal data using the WHOQOL-BREF instrument, implementing the best practices described in Rogers (2024):

1. **Data preparation** (Section 3): Loaded data directly from Mendeley Data repository, verified structure and descriptive statistics
2. **Population model** (Section 4): Defined a teaching example based on Lin & Yao (2022) with analytically derived residual variances and equidistant thresholds
3. **Model specification and estimation**: Tested four competing structures:
   - Original 4-factor model with all 24 items (Section 5)
   - Bifactor model with general QOL factor (Section 6)
   - Second-order hierarchical model (Section 7)
   - Revised 4-factor model excluding problematic item Q5 (Section 8)
4. **Model evaluation**: Applied DWLS estimation appropriate for ordinal data, examining overall fit (SRMR, RMSEA, CFI), local fit (MI, residuals), reliability (alpha, omega, AVE), and path diagrams
5. **Model comparison** (Section 9): Systematically compared all models, recommending the revised 4-factor model
6. **Power analysis** (Section 10): Assessed statistical adequacy via Monte Carlo simulation

❗ Key Findings

1. **Always use appropriate estimators for ordinal data**: DWLS/WLSMV operates on polychoric correlations and does not assume normality of observed variables — using ML with ordinal Likert data leads to biased estimates
2. **Item Q5 is systematically problematic** across multiple model structures and should be excluded
3. **The revised 4-factor correlated model** (without Q5) provides the best balance of fit, interpretability, and statistical properties
4. **Bifactor models improve fit mechanically** but impose strong orthogonality assumptions that rarely hold; they should not be preferred solely on the basis of fit indices
5. **All four domains** show adequate reliability (omega > .70) in the recommended model
6. **Sample size N = 1047** provides adequate power for all parameters of substantive interest
7. **Examine both overall and local fit** — global indices can mask specific, actionable problems
8. **Modification indices should guide, not dictate**: Only make theoretically justified changes, and be alert to items that appear repeatedly in high MIs

For the full theoretical and methodological framework underlying these analyses, including parallel JASP implementations, see Rogers (2024), OSF repository, and Github Repository

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